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1. Introduction.

As the title suggests, this book is intended to provide an introduction to modern set theory, to readers with little or no knowledge of mathematical logic. As such, it should be useful to anyone interested in learning about modern set theory, without having to wade through an entire text such as the “Millennium Edition” [Jech2]. Readers might fall in to two categories, those who are not interested in reading further, and those who are. For the latter, this book hopefully provides useful orientation.

It is hoped that advanced high school students will find this book useful. Admittedly only the most intrepid student would finish it in high school; but the first 15 chapters, and the two appendices, are hopefully fairly accessible. Resources for advanced high school mathematics are mainly in calculus and linear algebra, with some resources in other areas. Resources in mathematical logic have typically been scarce, one example being a 1958 book on Godel’s proof [NagNew]. The website [Wiki, Mathematical logic] has overviews of various topics, and links to additional resources.

The present book contains an introduction to mathematical logic sufficient for its purposes, and thus should serve as a useful introduction for other purposes. Various other topics are covered for the same reason, so that the book is fairly self-contained.

Set theory, like any branch of contemporary mathematics, consists of an overwhelming volume of technical definitions and arguments. On the other hand, non-technical introductions sometimes engage in circumlocutions intended to avoid technical detail, so convoluted that they become confusing. The present book pursues an intermediate course, covering technical details in outline and giving references, so that the main content can be given with some discussion of technical details.

The book consists of a series of sections, each covering a particular topic. The table of contents gives a list of the sections. The end of a proof is denoted using the symbol “◁”. The author thanks Dr. Herbert Enderton for reading a draft of the manuscript.